

Puzzle Pieces

Andy is putting a puzzle together. The puzzle has one hundred fifty pieces all together. The first day Andy puts thirty-six pieces of the puzzle together. The second day Andy puts forty-one more pieces of the puzzle together. The third day Andy puts sixty-eight more pieces of the puzzle together. Andy is upset because there are no more puzzle pieces left. How many puzzle pieces does Andy need to finish the puzzle? Show all your mathematical thinking.

Puzzle Pieces

Common Core Task Alignments

Mathematical Practices: 1, 3, 4, 6,

Grade 2 Content Standards:
2.NBT.B.6

Task

Andy is putting a puzzle together. The puzzle has one hundred fifty pieces all together. The first day Andy puts thirty-six pieces of the puzzle together. The second day Andy puts forty-one more pieces of the puzzle together. The third day Andy puts sixty-eight more pieces of the puzzle together. Andy is upset because there are no more puzzle pieces left. How many puzzle pieces does Andy need to finish the puzzle? Show all your mathematical thinking.

Common Core Content Standards and Evidence

2.NBT Number and Operations in Base Ten

Use place value understanding and properties of operations to add and subtract.

6. Add up to four two-digit numbers using strategies based on place value and properties of operations.

Exemplars Task-Specific Evidence

This task requires students to use properties of operations and place value to add three two-digit numbers.

Underlying Mathematical Concepts

- Addition/Subtraction
- Part/Whole reasoning
- Number sense to 150
- Ordinal numbers

Possible Problem-Solving Strategies

- Model (manipulatives)
- Diagram/Key
- Table
- Number line

Possible Mathematical Vocabulary/Symbolic Representation

- Model
- Diagram/Key
- Number line
- Amount
- Table
- Ordinal numbers (1st, 2nd, 3rd ...)
- Difference

Possible Mathematical Vocabulary/Symbolic Representation (cont.)

- Total/Sum
- Odd/Even
- More than (>)/Greater than (>)/Less than (<)
- Equivalent/Equal to
- Most/Least
- Per
- Number Line

Possible Solutions

Andy needs 5 more puzzle pieces.

Day	Pieces	Total Pieces
1	36	36
2	41	77
3	68	145

$$36 + 41 = 77$$

$$77 + 68 = 145$$

$$150 - 145 = 5$$

36	77
+ 41	+ 68
70	130
+ 7	+ 15
77	145

Possible Connections

Below are some examples of mathematical connections. Your students may discover some that are not on this list.

- Each day Andy puts a greater total of pieces in the puzzle.
- 41 puzzle pieces is an odd number.
- 36 puzzle pieces is 3 dozen.
- 68 puzzle pieces were the most done in a day.
- 36 puzzle pieces were the least done in a day.
- Solve more than one way to verify the answer.
- Relate to a similar problem and state a math link.
- On day 1, Andy puts an even number of pieces together.
- On day 2, Andy puts an odd number of pieces together.
- 50 pieces per day would have been an equal amount per day.
- Even + Odd = Odd, so it is clear some pieces are missing.

Novice Scoring Rationales

Criteria and Performance Level	Assessment Rationales
Problem Solving <i>Novice</i>	It appears that the student is adding all the numbers located in the problem for a total of 285. This strategy does not solve the problem. The student's answer, "285 pieces," is not correct.
Reasoning Proof <i>Novice</i>	The student does not demonstrate correct reasoning. The student does not understand that the placed puzzle pieces need to be totaled and subtracted from 150 pieces to determine how many pieces are needed to finish the puzzle.
Communication <i>Novice</i>	The student does not use any mathematical language to communicate her/his reasoning and proof. Verbs, such as "add," are not included in considering communication as they are considered an action the student takes in her/his solution.
Connections <i>Novice</i>	The student does not make a mathematically relevant observation about her/his solution.
Representation <i>Novice</i>	The student does not attempt a mathematical representation to solve or portray her/his solution.

Novice

P/S	R/P	Com	Con	Rep	A/Level
N	N	N	N	N	N

I want to find how many pieces Andy needs to find. I will add.

$$\begin{array}{r} 150 \\ 36 \\ 41 \\ + 68 \\ \hline 285 \end{array}$$

A ✖
285 pieces

Andy has to find a lot of pieces.

Apprentice Scoring Rationales

Criteria and Performance Level	Assessment Rationales
Problem Solving <i>Apprentice</i>	The student has a partially correct strategy. The student totals the puzzle pieces used in the three days. The student does not find the difference between the puzzle pieces used and the original 150 puzzle pieces. The student's answer, "145 pieces of puzzle," is not correct.
Reasoning Proof <i>Apprentice</i>	The student uses some correct reasoning of the underlying concepts of the problem. The student organizes and determines the 145 puzzle pieces used in three days. The student does not determine how many puzzle pieces are needed to finish the puzzle.
Communication <i>Practitioner</i>	The student correctly uses the mathematical term <i>day</i> , from the problem. The student also correctly uses the mathematical terms <i>table</i> , <i>odd</i> , <i>total</i> .
Connections <i>Practitioner</i>	The student makes the mathematically relevant observation, "41 is only odd pieces total in a day."
Representation <i>Practitioner</i>	The student's table is appropriate to the task and accurate. All labels are included and the data is correct.

Apprentice

P/S	R/P	Com	Con	Rep	A/Level
A	A	P	P	P	A

I need to find how many pieces are lost.

I will make a table.

day	Pieces	
1	36	36.
2	41	41.
3	68	+68.
		<u>145</u>

A
145 pieces of puzzle

41 is only odd pieces total in a day

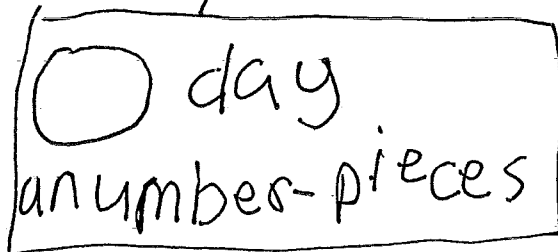
Practitioner Scoring Rationales, Student 1

Criteria and Performance Level	Assessment Rationales
Problem Solving <i>Practitioner</i>	The student's strategy of making a diagram of the puzzle pieces used each of three days works to solve part of the problem. The student's answer, "5," is correct.
Reasoning Proof <i>Practitioner</i>	The student uses correct reasoning of the underlying concepts of the problem. The student organizes and determines a total of 145 puzzle pieces are used in three days. The student finds the number of remaining puzzle pieces needed to finish the puzzle by applying subtraction.
Communication <i>Practitioner</i>	The student correctly uses the mathematical term <i>day</i> from the problem. The student also correctly uses the mathematical terms <i>diagram</i> , <i>key</i> , <i>number</i> .
Connections <i>Practitioner</i>	The student makes the mathematically relevant observations, "I see 36 is 3 dozen pieces" and "I see 68 is the most pieces." The student's statement, "The 5 pieces are on the floor," is not considered a mathematically relevant statement. This comment could lead to an engaging classroom writing activity determining where those five missing pieces are.
Representation <i>Practitioner</i>	The student's diagram is appropriate to the task and accurate. A key defines the days and puzzle pieces. The entered "numbers" are correct.

Practitioner, Student 1

P/S	R/P	Com	Con	Rep	A/Level
P	P	P	P	P	P

I have to find out how many puzzle pieces Andy needs. I will make a diagram and a key.



$$\begin{array}{r}
 150 \\
 -145 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 36 + 41 + 68 \\
 30 + 40 + 60 = 130 \\
 6 + 1 + 8 = 15
 \end{array}$$

○ answer $130 + 15 =$
 $100 + 40 + 5 = 145$

I see 36 is 3 dozen pieces.
 I see 68 is the most pieces.
 The 5 pieces are on the floor.

Practitioner Scoring Rationales, Student 2

Criteria and Performance Level	Assessment Rationales
Problem Solving <i>Expert</i>	<p>The student's strategy of making a table of the day, puzzle pieces and total puzzle pieces works to solve part of the problem and arrives at an answer by applying subtraction. The student's answer, "the answer is 5 puzzle pieces," is correct. The student shows evidence of analyzing the situation in mathematical terms. The student explores the numbers in the ones place and determines that a total of 150 pieces is not possible.</p>
Reasoning Proof <i>Expert</i>	<p>The student demonstrates correct reasoning of the underlying concepts of the problem. The student organizes and determines a total of 145 puzzle pieces are used in three days. The student finds the number of remaining puzzle pieces needed to finish the puzzle by applying subtraction. The student explains the phenomenon of how some sets of numbers can not have a total with a zero in the ones place. The student uses that thinking to explain how she/he realizes that five puzzle pieces must be missing.</p>
Communication <i>Practitioner</i>	<p>The student correctly uses the mathematical term <i>day</i>, from the problem. The student also correctly uses the mathematical terms <i>ones place</i>, <i>dozen</i>.</p>
Connections <i>Expert</i>	<p>The student makes the Practitioner connection, "I know 36 is $12 + 12 + 12$, 3 dozen." The student makes an Expert connection by extending her/his understanding of this problem to place value. "Look at the ones place $36 + 41 + 68$, $6 + 1 + 8 = 15$. It can't be 150. So + 5 pieces gives the zero. $36 + 41 + 68 = 145$, $145 + 5 = 150$."</p>
Representation <i>Practitioner</i>	<p>The student's table is appropriate to the task and accurate. All labels are included and the data is correct.</p>

Practitioner, Student 2

P/S	R/P	Com	Con	Rep	A/Level
E	E	P	E	P	P

Find how many pieces are missing. I can do a plan.

The puzzle
the day | the puzzle pieces | used pieces

1	36	36
2	41	77
3	68	145

$$\begin{array}{r} 36 \\ + 41 \\ \hline 77 \end{array}$$

$$\begin{array}{r} 77 \\ + 68 \\ \hline 145 \end{array}$$

$$\begin{array}{r} 150 \\ - 145 \\ \hline 5 \end{array}$$

the answer is 5 puzzle pieces

Practitioner, Student 2 (cont.)

look at the ones place

$$3(6) + 4(1) + 6(8)$$

$$6 + 1 + 8 = 15$$

It can't be 15(0)

so + (5) pieces gives the

$$\text{zero } 3(6) + 4(1) + 6(8) = 145$$

$$+ 5$$

$$\hline 150$$

I know 36 is $\underline{12} + \underline{12} + \underline{12}$
3 dozen

Expert Scoring Rationales

Criteria and Performance Level	Assessment Rationales
Problem Solving <i>Expert</i>	<p>The student’s strategy of making a table of the days, puzzle pieces and total puzzle pieces works to solve part of the problem. The student applies subtraction and states a correct answer, “5 pieces.” The student shows evidence of analyzing the situation in mathematical terms. The student explores the numbers in the ones place and determines that a total of 150 pieces is not possible. The student also applies the Even + Odd + Even = Odd rule to support her/his analysis. The student also verifies that her/his solution is correct by using number lines as a new strategy.</p>
Reasoning Proof <i>Expert</i>	<p>The student uses correct reasoning of the underlying concepts of the problem. The student organizes and determines a total of 145 puzzle pieces are used in three days. The student finds the number of remaining puzzle pieces needed to finish the puzzle by applying subtraction. The student explains the phenomenon that when adding, some sets of numbers can not have a total with a zero in the ones place. The student uses this thinking to explain how she/he realizes that some puzzle pieces must be missing. The student uses number lines to verify that the data in her/his table is correct.</p>
Communication <i>Expert</i>	<p>The student correctly uses the mathematical term <i>day</i> from the problem. The student also correctly uses the mathematical terms <i>table, 1st, 2nd, 3rd, total, dozen, most, number line, odd, even, sets, rule</i>. The student correctly uses the symbolic notation $E + O + E = O$ and defines E and O under the numbers.</p>

Expert Scoring Rationales (cont.)

Criteria and Performance Level	Assessment Rationales
<p>Connections <i>Expert</i></p>	<p>The student makes the mathematically relevant Practitioner observation, "36 is a dozen pieces" and "He puts the most pieces in on the 3rd day." The student makes the Expert connection, "36 41 68, $6 + 1 + 8 = 15$. The sum does not end in 0 or 5 so they can not add up to 150." The student also states $E + O + E = O$, $36 + 41 + 68 = 145$, even + odd + even = odd." The student applies this rule by using the equations, $6 + 1 + 8 = 15$, $8 + 5 + 2 = 15$, $4 + 9 + 10 + 23$, and states, "it is a true rule." The student also verifies her/his answer by using two number lines to arrive at 150 pieces. The first number line considers addition and the second number line considers subtraction. The student states, "I am correct. I found 5 pieces 3 ways. I thought of all I know."</p>
<p>Representation <i>Expert</i></p>	<p>The student's table is appropriate to the task and accurate. All labels are included and the data is correct. The student attempts a number line and discards it as she/he realizes the paper is too short to have a number line using increments of five. The student makes two correct number lines with increments of ten. The student indicates that the numbers represent pieces. The student compares these two number lines with her/his table to verify that her/his answer is correct.</p>

Expert

P/S	R/P	Com	Con	Rep	A/Level
E	E	E	E	E	E

I need to find out how many Andy has missing in the puzzle.
I will make a table.

days	puzzle pieces used	total pieces
1st	36	36
2nd	41	77
3rd	68	145

$$\begin{array}{r} 36 \\ 41 \\ \hline 77 \end{array}$$

$$\begin{array}{r} 77 \\ +68 \\ \hline 145 \end{array}$$

$$150 - 145 = 5$$

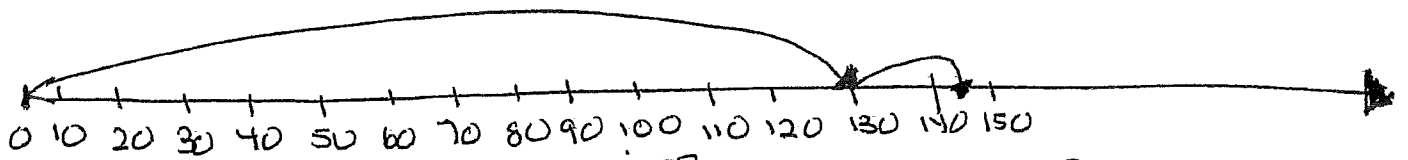
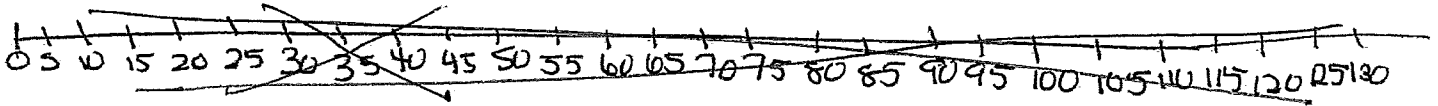
Answer
5 pieces

36 41 68
6 + 1 + 8 = 15
The sum does not end in 0 or 5 so they cannot add up to 150

36 is a dozen pieces.
He puts the most pieces in on the 3rd day.

Expert (cont.)

I can to a number line.

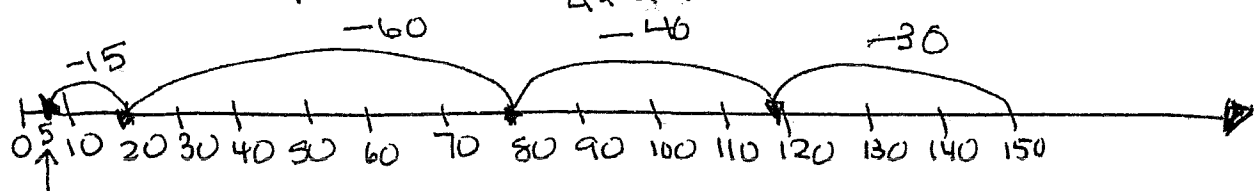


odd - 36 do 1st do 2nd
 odd - 41 30 + 6
 even - 68 40 + 1
 + 60 + 8
 130 15
 145

$E + O + E = O$
 $36 + 41 + 68 = 145$
 even + odd + even = odd
 $6 + 1 + 8 = 15$
 $8 + 5 + 2 = 15$ it is true rule
 $4 + 9 + 6 = 23$
 -46

$145 + 5 = 150$

this is add sets of pieces



36 30 + 6
 41 40 + 1
 68 + 60 + 8
 130 15
 ✓
 145

5 left

this is subtract sets of pieces

I am corect.
 IF you d 5 pieces
 3WAYS, I thinked of
 all I know.